# On space-time noncommutative U(1) model at high temperature[1]

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**Abstract**: We extend the results of Ref. [1] to noncommutative gauge theories at finite temperature. In particular, by making use of the background field method, we analyze renormalization issues and the high-temperature asymptotics of the one-loop Euclidean free energy of the noncommutative $U(1)$ gauge model within imaginary time formalism. As a by-product, the heat trace of the non-minimal photon kinetic operator on noncommutative $S^1 \times R^3$ manifold taken in an arbitrary background gauge is investigated. All possible types of noncommutativity on $S^1 \times R^3$ are considered. It is demonstrated that the non-planar sector of the model does not contribute to the free energy of the system at high temperature. The obtained results are discussed.

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## 1 Introduction

Understanding fundamental properties of hot plasma in noncommutative gauge theories, especially in NC QED, remains one of the most challenging problems in high-energy physics. Indeed, because of the noncommutative nature of space-time, even the simplest thermal $U(1)$ model exhibits such odd features as generation of the magnetic mass (associated with noncommutative transverse modes), appearance of a tachyon in the spectrum of quasi-particle excitations etc. [2, 3, 4, 5, 6, 7, 9, 10]. These observations concern mainly space/space noncommutative theories where there are no notorious difficulties with causality and unitarity [11, 12]. At the same time, it was realized that a space/space NC QFT may have non-renormalizable divergences as a consequence of UV/IR mixing phenomenon [13] (see also [14] for recent discussion).

The purpose of the present work is to gain some better insight into basic aspects of the Euclidean-time formalism in thermal gauge theories on NC $S^1 \times R^3$, including renormalization and the high-temperature asymptotic of the (Euclidean) free energy (FE). For the sake of completeness, three different types of noncommutative space-time will be worked out: namely, space/space, full-rank and pure space/time noncommutativities. We begin our analysis with the investigation of one-loop divergences in the Euclidean NC $U(1)$ gauge model on $S^1 \times R^3$ to make sure that the theory does exist at least at the leading order of the loop expansion. Then we will turn to the evaluation of the high-temperature asymptotics of the one-loop FE. The main attention will be paid to the non-planar sector of the perturbative expansion. Thus, it was discovered in Ref. [4, 5] that there is a drastic reduction of the degrees of freedom in non-planar part of FE. Here we will arrive at the same qualitative picture for all types of noncommutativity.

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## 2 The model

Consider $U(1)$ gauge model on NC $S^1 \times R^3$. Its action reads[2]

$$S = -\frac{1}{4\,g^2} \int_{\mathcal{M}} d^4x \; G_{\mu\nu} \star G_{\mu\nu}, \tag{1}$$

where the integration is carried out over $\mathcal{M} = S^1 \times R^3$ manifold and $G_{\mu\nu}$ denotes the curvature tensor of $U(1)$ gauge connection.

To investigate quantum corrections to (1) we employ the background field method. To this aim we split the field $A_\mu$ into a classical background field $B_\mu$ and quantum fluctuations $Q_\mu$, i.e. $A_\mu = B_\mu + Q_\mu$. Then, substituting this decomposition into (1), we extract the part of the action (1) that is quadratic in quantum fluctuations. In a covariant background gauge it is written in the form (we use notations of Ref. [27]):

$$S_2[B, Q, \overline{C}, C] = \int_{\mathcal{M}} d^4x \; \left( -\frac{1}{2g^2} Q_\mu(x) D^{(\xi)}_{\mu\nu} Q_\nu(x) + \overline{C}(x) \; DC(x) \right), \tag{2}$$

where

$$D^{\xi}_{\mu\nu} = -\Big[\delta_{\mu\nu}\nabla^2 + (\frac{1}{\xi} - 1)\nabla_\mu\nabla_\nu + 2(L_\Theta(F_{\mu\nu}) - R_\Theta(F_{\mu\nu}))\Big] \tag{3}$$

is the photon kinetic operator and $D = -\nabla_\mu\nabla_\mu$ is the inverse propagator of ghost particles. Here $\nabla_\mu$ and $F_{\mu\nu}$ stand for the covariant derivative and the curvature tensor of the background field $B_\mu$, respectively. Functional integration of the partition function w.r.t. quantum fields gives the following formal expression for the 1-loop effective action (EA),

$$\Gamma^{(1)}[B] = \Gamma_{gauge}[B] + \Gamma_{ghost}[B] = \frac{1}{2} \ln\det\left(D^{\xi}\right) - \ln\det(D). \tag{4}$$

As well-known this quantity is divergent and must be regularized. This will be done by zeta-function regularization in what follows.

For the study of thermal QFT one needs to introduce another important object – the free energy of the system. Recall, that there are two definitions of this quantity. One of them presents the canonical FE,

$$F^C(\beta) = \beta^{-1} \sum_{\omega} \ln\left(1 - e^{-\beta\omega}\right), \tag{5}$$

which has clear physical meaning of "summation over modes". The other one expresses FE in terms of the Euclidean EA,

$$F^E(\beta) = \beta^{-1}\Gamma^E(\beta), \tag{6}$$

and is much more convenient from practical point of view. These two definitions are related by

$$F^E(\beta) = F^C(\beta) + E_0,$$

where $E_0$ is the energy of vacuum fluctuations. It should be noted, however, that a rigorous proof of this relation even in conventional field theories may be a highly non-trivial task (e.g. for thermal systems in curved spaces, see for instance Refs. [15, 16]). The equivalence of the canonical and Euclidean FE in QFT with space-time noncommutativity (although with some heuristic assumptions) was discussed in Ref. [1].

## 3 Zeta-function regularization.

In the zeta regularization scheme, the regularized EA (4) is represented by [17, 18, 19]

$$\Gamma^{(1)}_s[B] = -\frac{1}{2}\mu^{2s}\Gamma(s)\left(\zeta\left(s, D^{\xi}\right) - 2\zeta(s, D)\right), \tag{7}$$

[2] As usual, we will work in the rest frame of the heat bath with $u = (0, 0, 0, 1)$, where $u$ is the heat bath four velocity. All fields obey periodic boundary conditions.

where $\zeta\left(s, D^{\xi}\right)$ and $\zeta\left(s, D\right)$ are zeta-functions of each operator in (4), $s$ is a renormalization parameter and $\mu$ is introduced to render the mass dimension correct. The regularization is removed in the limit $s \to 0$ giving

$$\Gamma^{(1)}_{s\to 0}[B] = -\frac{1}{2}\left(\frac{1}{s} - \gamma_E + \ln\mu^2\right)\zeta_{tot}(0) - \frac{1}{2}\zeta'_{tot}(s), \tag{8}$$

where $\gamma_E$ is the Euler constant and $\zeta_{tot}(s) = \zeta\left(s, D^{\xi}\right) - 2\zeta(s, D)$.

To deal with the zeta-functions we need to introduce the heat traces for the operators $D^{\xi}$ and $D$, respectively. Recall that for a star-differential operator $\mathcal{D}$ it is define as

$$K\left(t, \mathcal{D}\right) = \mathrm{Tr}_{L^2}\left(\exp(-t\mathcal{D}) - \text{volume term}\right), \tag{9}$$

where $t$ is the spectral (or "proper time") parameter. Symbol $\mathrm{Tr}_{L^2}$ denotes $L_2$-trace taken on the space of square integrable functions ( on $S^1 \times R^3$ with periodic boundary conditions in our case) and may also involve the trace over vector, spinor etc. indices. The main technical result here is that on a (flat) NC manifold the heat trace (9) can be expanded in power series in small $t$ as:

$$K(t, \mathcal{D}) = \sum_{n=1}^{\infty} t^{(n-4)/2} a_n(\mathcal{D}). \tag{10}$$

For further details, we refer the interested reader to Refs. [20, 21, 22, 23, 24, 1, 25]. Now, the zeta-function $\zeta_{tot}(s)$ has the following integral representation,

$$\zeta_{tot}(s) = \frac{1}{\Gamma(s)}\int_0^{\infty}\frac{dt}{t^{1-s}}\left(K^{\xi}\left(t, D^{\xi}\right) - 2K\left(t, D\right)\right), \tag{11}$$

and to analyze the structure of (8) one should actually evaluate the heat trace coefficients for each operator entering (4). For instance, taking into account the relation $a_k(\mathcal{D}) = \mathrm{Res}_{s=(4-k)/2}\Gamma(s)\zeta(s, \mathcal{D})$, the pole part of (8) can be re-expressed through the heat trace coefficients as

$$\Gamma^{(1)}_{\text{pole}}[B] = -\frac{1}{2}\left(\frac{1}{s} - \gamma_E + \ln\mu^2\right)\left(a_4(D^{\xi}) - 2a_4(D)\right). \tag{12}$$

That is, on a 4-dimensional manifold it is determined by the 4th heat trace coefficients.

# 4 Evaluation of the heat trace coefficients

To obtain the heat trace asymptotics of the non-minimal operator (9) it is convenient to use the calculating method by Endo [26] generalized on a NC case [27]. Namely, if the background field satisfies the equation of motion, the following relation holds[3]:

$$\begin{aligned} K^{\xi}\left(t, D^{(\xi)}\right) &= K^{\xi=1}\left(t, D^{(\xi=1)}\right) \\ &- \int_t^{\frac{t}{\xi}} d\tau \int_{\mathcal{M}} d^4x \left(\nabla_\mu \nabla'_\mu K(x, x'; \tau|\beta) - \text{volume term}\right)|_{x=x'}, \end{aligned} \tag{13}$$

where $K(x, x'; \tau|\beta)$ is the thermal heat operator of the inverse ghost propagator. Notice that RHS of this relation consists of the heat traces of *minimal* star-differential operators. Calculating procedure for such objects is standard and described, for instance, in Ref. [23]. In particular, it was found that the heat trace expansion for a generalized star-Laplacian[4] contains coefficients of two types: so-called planar and mixed heat trace coefficients. In our example, the first planar heat trace coefficient is given by

$$a_4^{\text{pl.tot.}} := a_4(D^{(\xi)}) - 2a_4(D) = \frac{1}{16\pi^2}\left(-\frac{11}{3}\right)\int_{\mathcal{M}} d^4x\, F_{\mu\nu} \star F_{\mu\nu}. \tag{14}$$

[3]Notice that one has to eliminate volume divergences by adding appropriate terms, cf. expr. (9).
[4]That is, which includes both left and right Moyal multiplications.

Evaluation of the mixed heat trace coefficients, however, is more involved. Here we inspect three different cases.
(i) Full-rank noncommutativity. To simplify computations we assume that the deformation matrix $\Theta$ has the canonical form:

$$\Theta = \begin{pmatrix} \theta S & 0 \\ 0 & \vartheta S \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{15}$$

However, the reader should be warned that, in general, a reference frame where the matrix $\Theta$ has the block off-diagonal form (15) does not necessarily coincide with the reference frame of the heat bath. The first nontrivial mixed coefficient can be now easily evaluated and has the form (see also [1] for some technical details)

$$\begin{aligned} a_5^{\text{mix.tot.}} &= -\frac{\xi^{-1/2}}{2\beta\,\theta^2\pi^{5/2}} \sum_{n\in Z} \int_{R^2\times S^1} dx_\perp dx^4 \int_{R^2\times S^1} dy_\perp dy^4 \int_R dx^3 \times \\ &\times \sum_{\mu,\,\mu\neq 3} B_\mu\left(x^1, x^2, x^3 + \frac{\pi|\vartheta|n}{\beta}; x^4\right) B_\mu\left(y^1, y^2, x^3 - \frac{\pi|\vartheta|n}{\beta}; y^4\right). \end{aligned} \tag{16}$$

This coefficient is divergent as $\theta \to 0$ and/or $\vartheta \to 0$ that is a manifestation of the well-known UV/IR phenomenon [28, 29, 30].
(ii) Pure space/time noncommutativity ($\Theta^{ij} = 0$ and $\Theta^{i4}$ is directed along $x_\parallel$ axis). In this case the first mixed heat trace coefficient is presented by

$$\begin{aligned} a_3^{\text{mix.tot.}} &= -\frac{1}{2\beta\,\pi^{3/2}} \left(2 - \sqrt{\xi}\right) \sum_{n\in Z} \int_{S^1\times S^1} dx^4 dy^4 \int_{R^3} dx_\perp\, dx_\parallel \times \\ &\times B_4\left(x_\perp, x_\parallel + \frac{\pi|\vartheta|n}{\beta}; x^4\right) B_4\left(x_\perp, x_\parallel - \frac{\pi|\vartheta|n}{\beta}; y^4\right). \end{aligned} \tag{17}$$

(iii) Space/space noncommutativity ($\Theta^{ij} \neq 0$, $\Theta^{4i} = 0$). One finds

$$\begin{aligned} a_4^{\text{mix.tot.}} &= \frac{(\ln\xi - 2)}{8\,\theta^2\pi^3} \int_{S^1\times R} dx^3 dx^4 \int_{R^2\times R^2} dx_\perp\; dy_\perp \times \\ &\times \sum_{i=1,2} B_i(x_\perp, x^3; x^4)\; B_i(y_\perp, x^3; x^4). \end{aligned} \tag{18}$$

From (12) we see that this coefficient does contribute to the pole term of the one-loop EA and, hence, affects renormalization of the model that will be explained in a moment.

# 5 Renormalization and high-temperature asymptotics

Let us now look a little more closely at the divergent part of EA (12). Clearly, in the case of noncommutative compact dimension it is defined solely by the planar heat trace coefficient (14). That is, the pole part of the one-loop EA has the form

$$\Gamma^{(1)}_{\text{pole}}[B] = -\frac{1}{2s} \int_{\mathcal{M}} d^4x \left(-(4\pi)^{-2} \frac{22}{6}\, F_{\mu\nu} \star F_{\mu\nu}\right), \tag{19}$$

leading thus to the standard renormalization group. We see that the source of the UV divergence in (8) is associated with the original four-dimensional field theory and this divergence is removed by ordinary renormalization at zero temperature. However, the situation changes drastically when the compact coordinate is commutative: in this particular case the expression (19) contains an additional term due to the mixed heat trace coefficient (18). Although this new term is also temperature independent, it brings

into EA a non-local and, moreover, gauge-fixing dependent divergence which cannot be eliminated by any renormalization prescription.

To obtain high-temperature asymptotics of the one-loop EA we rewrite (7) as

$$\Gamma_s^{(1)}[B] = \mu^{2s} \sum_{k=2} \int_0^\infty \frac{dt}{t^{3-s}} t^{\frac{k}{2}} \left( \left( -\frac{1}{2} a_k(D^\xi) + a_k(D) \right) + \right.$$
$$\left. +2 \sum_{n=1} e^{-\frac{\beta^2 n^2}{4t}} \left( -\frac{1}{2} a_k^{\text{planar}}(D^\xi) + a_k^{\text{planar}}(D) \right) \right), \tag{20}$$

where we retained all exponentially small terms in the planar sector as well. (They must be taken into account when the parameter $\beta$ is small). The evaluation of the planar part proceeds exactly as in the conventional thermal $SU(2)$ gluodynamics giving

$$S_{\text{tree}}[B] + \Gamma^{(1)}_{\text{planar}}[B] \simeq \left( -\frac{1}{4 g_R^2(T)} \int_{\mathcal{M}} d^4 x F_{\mu\nu} \star F_{\mu\nu} \right. \tag{21}$$
$$\left. + \sum_{k=6} \left( \frac{\beta}{2} \right)^{2k-4} \left( a_k^{\text{planar}}(D^\xi) - 2 a_k^{\text{planar}}(D) \right) \zeta(2k-4) \Gamma(k-2) \right),$$

from which one deduces high temperature behaviour of NC $U(1)$ effective coupling:

$$g_R^2(T) = g_R^2 \left( 1 + \frac{g_R^2}{4\pi^2} \frac{11}{3} \ln\left(T/T_0\right) \right)^{-1}. \tag{22}$$

It should be emphasized, however, that the formula (22) makes sense unless a compact dimension is commutative: as we have already seen, within space/space NC $U(1)$ model one cannot renormalize the charge because of the non-planar contribution (18).

Now consider the non-planar part of EA. For the sake of definiteness let us focus on the pure space/time noncommutativity. First of all, we note that the expression (17) is valid whenever the condition $|\vartheta|/\beta \neq 0$ holds. Hence, it is interesting to explore high temperature regime when $|\vartheta|/\beta \gg C_0, \quad C_0 \in R_+$. We assumed that the background field $B_\mu \in C^\infty(S^1 \times R^3)$ and, therefore, it should vanish exponentially fast at large distances. For $n \neq 0$ one estimates

$$B_\mu \left( x_1, x_2, z + \frac{\pi |\vartheta| n}{\beta}; x_4 \right) B_\mu \left( y_1, y_2, z - \frac{\pi |\vartheta| n}{\beta}; y_4 \right) \sim C_2 \exp\left( -C_1 \frac{|\vartheta|}{\beta} \right), \quad \frac{|\vartheta|}{\beta} \gg C_0,$$

where $C_1$ is a positive constant which characterizes the fall-off of the gauge potential at large distances. Up to an inessential overall constant the contribution of the first mixed coefficient to the effective potential can be estimated as

$$a_3^{\text{tot}} = \frac{(1+\sqrt{\xi})}{2\beta(\pi)^{3/2}} \int_{S^1 \times S^1} dx^4 dy^4 \int_{R^3} dx \, B_4\left(\bar{x}; x^4\right) \, B_4\left(\bar{x}; y^4\right). \tag{23}$$

Notice that this expression is insensitive to the value of the deformation parameter[5]. Moreover, since in the limit $\beta \to 0$ the main contribution to (23) comes from the zero bosonic modes, the mixed heat trace coefficients behave as $\sim \beta C$, where $C$ is some temperature-independent quantity. From the definition (6) it follows that, at least on the one-loop level, the non-planar part of EA provides the temperature-independent contribution to the Euclidean FE and therefore can be neglected in the high temperature limit.

[5]Of course, this does not mean that the expression (4) possesses a smooth commutative limit: in obtaining high-temperature asymptotics for (23) we assumed $|\vartheta| \neq 0$.

## 6 Conclusion

In this paper we have investigated the one-loop quantum corrections to EA (resp. Euclidean FE) in NC thermal $U(1)$ theory within the imaginary time formalism. Let us summarize the obtained results.

First, in the space/space noncommutative QED, the renormalizability of the theory is ruined by the non-planar sector of the perturbative expansion. This phenomenon was already observed, for instance, in Ref. [13] (see also [24, 27]). At the same time, in the case of a noncommutative compact dimension the theory can be renormalized, at least on one-loop level, by the standard renormalization prescription.

Second, we calculated the heat trace asymptotics for the non-minimal photon kinetic operator on NC $S^1 \times R^3$. We saw, in particular, that the noncommutativity of the compact coordinate results in arising of additional odd-numbered coefficients in the heat trace expansion. Furthermore, in the case of pure space/time noncommutativity the first nontrivial mixed contribution to the heat trace appears in $a_3^{\text{mixed}}$. Although this coefficient does not affect counterterms in the zeta function regularization, it can lead to certain troubles in different regularization schemes, see Ref.[1] for further discussion.

Third, we obtain the high-temperature asymptotics of the one-loop Euclidean FE (6). It is rather remarkable that the non-planar sector does not contribute at high temperature for any type of noncommutativity. This seems to be in accordance with observations made in earlier works where a drastic reduction of the degrees of freedom in non-planar part of FE was discovered [4, 5]. There is a subtlety, however, that one should keep in mind. Namely, if noncommutativity does not involve time, there are no difficulties in developing the Hamiltonian formalism for a NC theory and equivalence of the canonical and Euclidean free energies is proved by standard arguments [31]. Contrary to this, in the space/time NC theories there is no good definition of the canonical Hamiltonian and, consequently, of the canonical FE (5) although some progress in this direction has been made recently in Ref.[1].

Finally, an extension of our results to more general case of U(N) gauge symmetry can be done straightforwardly. Indeed, one can show that the mixed heat trace coefficients are completely determined by U(1) part of the model [27]. In the diagrammatic approach this implies the known fact that non-planar one-loop U(N) diagrams contribute only to the U(1) part of the theory [28, 32].

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